SADLER UNIT 3 CHAPTER 4

EXERCISE 4A

Q1.

$$\underset{\sim}{r}_A = \begin{pmatrix}5\\4\end{pmatrix} + t\begin{pmatrix}10\\-1\end{pmatrix}$$
$$= (5+10t)\underset{\sim}{i} + (4-t)\underset{\sim}{j}$$

$$\underset{\sim}{r}_B = \begin{pmatrix}6\\-8\end{pmatrix} + t\begin{pmatrix}2\\8\end{pmatrix}$$
$$= (6+2t)\underset{\sim}{i} + (8t-8)\underset{\sim}{j}$$

$$\underset{\sim}{r}_C = \begin{pmatrix}2\\3\end{pmatrix} + t\begin{pmatrix}-4\\3\end{pmatrix}$$
$$= (2-4t)\underset{\sim}{i} + (3+3t)\underset{\sim}{j}$$

$$\underset{\sim}{r}_D = \begin{pmatrix}9\\-10\end{pmatrix} + (t+1)\begin{pmatrix}10\\6\end{pmatrix}$$
$$= (9+10t+10)\underset{\sim}{i} + (-10+6t+6)\underset{\sim}{j}$$
$$= (19+10t)\underset{\sim}{i} + (6t-4)\underset{\sim}{j}$$

$$\underset{\sim}{r}_E = \begin{pmatrix}16\\7\end{pmatrix} + (t-1)\begin{pmatrix}-4\\3\end{pmatrix}$$
$$= (16-4t+4)\underset{\sim}{i} + (7+3t-3)\underset{\sim}{j}$$
$$= (20-4t)\underset{\sim}{i} + (4+3t)\underset{\sim}{j}, \quad t \geq 1$$

$$\underset{\sim}{r}_F = \begin{pmatrix}2\\3\end{pmatrix} + (t-0.5)\begin{pmatrix}12\\-8\end{pmatrix}$$
$$= (2+12t-6)\underset{\sim}{i} + (3-8t+4)\underset{\sim}{j}$$
$$= (-4+12t)\underset{\sim}{i} + (7-8t)\underset{\sim}{j}, \quad t \geq 0.5$$

Q2 $\underset{\sim}{r}_S = \begin{pmatrix}7\\10\end{pmatrix} + t\begin{pmatrix}3\\4\end{pmatrix}$

a) $\underset{\sim}{r}_S(1) = \begin{pmatrix}7\\10\end{pmatrix} + \begin{pmatrix}3\\4\end{pmatrix}$
$= 10\underset{\sim}{i} + 14\underset{\sim}{j}$

b) $\underset{\sim}{r}_S(2) = \begin{pmatrix}7\\10\end{pmatrix} + \begin{pmatrix}6\\8\end{pmatrix}$
$= 13\underset{\sim}{i} + 18\underset{\sim}{j}$

c) $\underset{\sim}{r}_S(4) = \begin{pmatrix}7\\10\end{pmatrix} + \begin{pmatrix}12\\16\end{pmatrix}$
$= 19\underset{\sim}{i} + 26\underset{\sim}{j}$

d) $|\underset{\sim}{v}_S| = \sqrt{9+16}$
$= 5$ km/hr

e) $\underset{\sim}{r}_S(3) = \begin{pmatrix}7\\10\end{pmatrix} + \begin{pmatrix}9\\12\end{pmatrix}$
$= \begin{pmatrix}16\\22\end{pmatrix}$

$\underset{\sim}{r}_S(3) - \begin{pmatrix}21\\20\end{pmatrix} = \begin{pmatrix}-5\\2\end{pmatrix}$

$\therefore \left|\begin{pmatrix}-5\\2\end{pmatrix}\right| = \sqrt{25+4}$
$= \sqrt{29}$ km.
$= 5.39$ km (2dp)

Q3. $\underset{\sim}{r}_S = \begin{pmatrix}9\\36\end{pmatrix} + t\begin{pmatrix}2\\12\end{pmatrix}$

$\underset{\sim}{r}_S(-1) = \begin{pmatrix}9\\36\end{pmatrix} - \begin{pmatrix}2\\12\end{pmatrix}$
$= \begin{pmatrix}7\\24\end{pmatrix}$
$= 7\underset{\sim}{i} + 24\underset{\sim}{j}$

a) $\left|\begin{pmatrix}7\\24\end{pmatrix}\right| = \sqrt{49+576}$
$= 25$ km

b) $\underset{\sim}{r}_S(-2) = \begin{pmatrix}9\\36\end{pmatrix} - 2\begin{pmatrix}2\\12\end{pmatrix}$
$= \begin{pmatrix}5\\12\end{pmatrix}$

$\left|\begin{pmatrix}5\\12\end{pmatrix}\right| = \sqrt{25+144}$
$= 13$ km

Q4. $\underset{\sim}{r}_A = \begin{pmatrix}21\\7\end{pmatrix} + t\begin{pmatrix}10\\5\end{pmatrix}$

$\underset{\sim}{r}_B = \begin{pmatrix}25\\-6\end{pmatrix} + t\begin{pmatrix}7\\10\end{pmatrix}$

$\underset{\sim}{r}_B - \underset{\sim}{r}_A = \begin{pmatrix}4\\-13\end{pmatrix} + t\begin{pmatrix}-3\\5\end{pmatrix}$

a) $|\underset{\sim}{r}_B - \underset{\sim}{r}_A| = \left|\begin{pmatrix}4\\-13\end{pmatrix}\right|$
$= \sqrt{16+169}$
$= \sqrt{185}$
$= 13.60$ km

b) $|\underset{\sim}{r}_B - \underset{\sim}{r}_B| = \left|\begin{pmatrix}4\\-13\end{pmatrix} + \begin{pmatrix}-3\\5\end{pmatrix}\right|$
$= \left|\begin{pmatrix}1\\-8\end{pmatrix}\right|$
$= \sqrt{1+64}$
$= \sqrt{65}$
$= 8.06$ km

c) $|\underset{\sim}{r}_B - \underset{\sim}{r}_A| = \left|\begin{pmatrix}4\\-13\end{pmatrix} + \begin{pmatrix}-6\\10\end{pmatrix}\right|$
$= \left|\begin{pmatrix}-2\\-3\end{pmatrix}\right|$
$= \sqrt{4+9}$
$= \sqrt{13}$
$= 3.61$ km

Q5 $\underset{\sim}{r}_A = \begin{pmatrix}-5\\13\end{pmatrix} + t\begin{pmatrix}7\\-2\end{pmatrix}$

$\underset{\sim}{r}_B = \begin{pmatrix}0\\-3\end{pmatrix} + t\begin{pmatrix}-3\\2\end{pmatrix}$

$\underset{\sim}{r}_B - \underset{\sim}{r}_A = \begin{pmatrix}5\\-16\end{pmatrix} + t\begin{pmatrix}-10\\4\end{pmatrix}$

a) $|\underset{\sim}{r}_B - \underset{\sim}{r}_A| = \left|\begin{pmatrix}5\\-16\end{pmatrix} + \begin{pmatrix}-10\\4\end{pmatrix}\right|$
$= \left|\begin{pmatrix}-5\\-12\end{pmatrix}\right|$
$= \sqrt{25+144}$
$= 13$ km.

b) $|r_B - r_A| = \left|\begin{pmatrix}5\\-16\end{pmatrix} + 2\begin{pmatrix}-10\\4\end{pmatrix}\right|$

$= \left|\begin{pmatrix}-15\\-8\end{pmatrix}\right|$

$= \sqrt{225+64}$

$= \sqrt{289}$

$= \underline{17 \text{ km}}$

Q6. $\underset{\sim}{r}_A = \begin{pmatrix}28\\-5\end{pmatrix} + t\begin{pmatrix}-8\\4\end{pmatrix}$

$r_B = \begin{pmatrix}0\\24\end{pmatrix} + t\begin{pmatrix}6\\2\end{pmatrix}$

a) $\therefore \underset{\sim}{r}_A = (28-8t)\underset{\sim}{i} + (4t-5)\underset{\sim}{j}$

$r_B = 6t\underset{\sim}{i} + (24+2t)\underset{\sim}{j}$

b) $r_B - r_A = \begin{pmatrix}-28\\29\end{pmatrix} + t\begin{pmatrix}14\\-2\end{pmatrix}$

$|r_B - r_A| = \left|\begin{pmatrix}14t-28\\29-2t\end{pmatrix}\right|$

$25 = \sqrt{(14t-28)^2 + (29-2t)^2}$

Using CAS to solve;

$t = 2$ and $t = 2.5$

$\therefore$ 25 km apart at 10am and 10:30am.

Q7. $r_A = \begin{pmatrix}12\\61\end{pmatrix} + t\begin{pmatrix}7\\-8\end{pmatrix}$

$r_B = \begin{pmatrix}57\\-29\end{pmatrix} + t\begin{pmatrix}-2\\10\end{pmatrix}$

$\begin{pmatrix}12+7t\\61-8t\end{pmatrix} = \begin{pmatrix}57-2t\\10t-29\end{pmatrix}$

Equating $\underset{\sim}{i}$ components:

$12+7t = 57-2t$

$9t = 45$

$\underline{t = 5}$

Equating $\underset{\sim}{j}$ components:

$61-8t = 10t-29$

$90 = 18t$

$\underline{t = 5}$

$\therefore$ Collision occurs at $\underline{1:00\text{pm}}$.

$\therefore \underset{\sim}{r}_A = r_B = \begin{pmatrix}12+35\\61-40\end{pmatrix} = \underline{\begin{pmatrix}47\\21\end{pmatrix}} \text{ km}$

Q8. $\underset{\sim}{r}_A = \begin{pmatrix}-11\\-8\end{pmatrix} + t\begin{pmatrix}7\\-1\end{pmatrix}$

$r_B = \begin{pmatrix}-2\\-4\end{pmatrix} + t\begin{pmatrix}4\\5\end{pmatrix}$

$\begin{pmatrix}-11+7t\\-8-t\end{pmatrix} = \begin{pmatrix}-2+4t\\-4+5t\end{pmatrix}$

Equating $\underset{\sim}{i}$ components:

$-11+7t = -2+4t$

$3t = 9$

$\underline{t = 3}$

Equating $\underset{\sim}{j}$ components:

$-8-t = -4+5t$

$-4 = 6t$

$\underline{t = -\frac{2}{3}}$

$\therefore$ No collision.

Q9. $r_A = \begin{pmatrix}24\\-25\end{pmatrix} + t\begin{pmatrix}-3\\4\end{pmatrix}$

$r_B = \begin{pmatrix}-9\\33\end{pmatrix} + (t-1)\begin{pmatrix}2\\-5\end{pmatrix}$

$\begin{pmatrix}24-3t\\4t-25\end{pmatrix} = \begin{pmatrix}-9+2t-2\\33-5t+5\end{pmatrix}$

$\begin{pmatrix}24-3t\\4t-25\end{pmatrix} = \begin{pmatrix}-11+2t\\38-5t\end{pmatrix}$

Equating $\underset{\sim}{i}$ components:

$24-3t = -11+2t$

$-5t = -35$

$\underline{t = 7}$

Equating $\underset{\sim}{j}$ components:

$4t-25 = 38-5t$

$9t = 63$

$\underline{t = 7}$

$\therefore$ collision occurs at $\underline{3:00\text{pm}}$

$\underset{\sim}{r}_A = r_B = \begin{pmatrix}24-21\\28-25\end{pmatrix} = \underline{\begin{pmatrix}3\\3\end{pmatrix}} \text{ km}$

Q10. $\underset{\sim}{r}_A = \begin{pmatrix}-6\\44\end{pmatrix} + t\begin{pmatrix}4\\-6\end{pmatrix}$

$\underset{\sim}{r}_B = \begin{pmatrix}2\\-18\end{pmatrix} + (t+0.5)\begin{pmatrix}2\\7\end{pmatrix}$

$$\begin{pmatrix}-6+4t\\44-6t\end{pmatrix} = \begin{pmatrix}2+2t+1\\-18+7t+3.5\end{pmatrix}$$

$$\begin{pmatrix}-6+4t\\44-6t\end{pmatrix} = \begin{pmatrix}3+2t\\-14.5+7t\end{pmatrix}$$

Equating $\underset{\sim}{i}$ components:

$-6+4t = 3+2t$

$2t = 9$

$\underline{t = 4.5}$

Equating $\underset{\sim}{j}$ components:

$44-6t = -14.5+7t$

$-13t = -58.5$

$\underline{\underline{t = 4.5}}$

$\therefore$ Collision occurs at 2pm

$$\underset{\sim}{r}_A = \underset{\sim}{r}_B = \begin{pmatrix}-6+18\\44-27\end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix}12\\17\end{pmatrix}\text{ km}}}$$

Q11. $\underset{\sim}{r}_A = \begin{pmatrix}-11\\4\end{pmatrix} + t\begin{pmatrix}10\\-4\end{pmatrix}$

$\underset{\sim}{r}_B = \begin{pmatrix}3\\-5\end{pmatrix} + (t-0.5)\begin{pmatrix}7\\5\end{pmatrix}$

$$\begin{pmatrix}-11+10t\\4-4t\end{pmatrix} = \begin{pmatrix}3+7t-3.5\\-5+5t-2.5\end{pmatrix}$$

$$\begin{pmatrix}-11+10t\\4-4t\end{pmatrix} = \begin{pmatrix}-0.5+7t\\-7.5+5t\end{pmatrix}$$

Equating $\underset{\sim}{i}$ components:

$-11+10t = -0.5+7t$

$3t = 10.5$

$\underline{\underline{t = 3.5}}$

Equating $\underset{\sim}{j}$ components:

$4-4t = -7.5+5t$

$-9t = -11.5$

$\underline{\underline{t = 1.2\dot{7}}}$

$\therefore$ No collision.

Q12. $\underset{\sim}{r}_P = \begin{pmatrix}-23\\3\end{pmatrix} + t\begin{pmatrix}18\\4\end{pmatrix}$

$\underset{\sim}{r}_P = \begin{pmatrix}-23+18t\\3+4t\end{pmatrix}$

$\underset{\sim}{r}_Q = \begin{pmatrix}7\\30\end{pmatrix} + t\begin{pmatrix}12\\-10\end{pmatrix}$

$\underset{\sim}{r}_Q = \begin{pmatrix}7+12t\\30-10t\end{pmatrix}$

$\underset{\sim}{r}_R = \begin{pmatrix}32\\-30\end{pmatrix} + t\begin{pmatrix}2\\14\end{pmatrix}$

$\underset{\sim}{r}_R = \begin{pmatrix}32+2t\\-30+14t\end{pmatrix}$

a)

Check P & Q:

$$\begin{pmatrix}-23+18t\\3+4t\end{pmatrix} = \begin{pmatrix}7+12t\\30-10t\end{pmatrix}$$

$-23+18t = 7+12t$

$6t = 29$

$\underline{t = 4.8\dot{3}}$

$3+4t = 30-10t$

$14t = 27$

$\underline{\underline{t = 1.93}}$ $\therefore$ Do not collide.

Check P & R:

$$\begin{pmatrix}-23+18t\\3+4t\end{pmatrix} = \begin{pmatrix}32+2t\\-30+14t\end{pmatrix}$$

$-23+18t = 32+2t$

$16t = 55$

$\underline{t = 3.44}$

$3+4t = -30+14t$

$-10t = -33$

$\underline{t = 3.3}$ $\therefore$ Do not collide

Check Q & R:

$$\begin{pmatrix}7+12t\\30-10t\end{pmatrix} = \begin{pmatrix}32+2t\\-30+14t\end{pmatrix}$$

$7+12t = 32+2t$

$10t = 25$

$\underline{t = 2.5}$

$30-10t = -30+14t$

$-24t = -60$

$\underline{\underline{t = 2.5}}$

$\therefore$ Boats Q & R collide when $t = 2.5$ (i.e at 10:30 am)

$$\underset{\sim}{r}_Q = \begin{pmatrix} 7 + 12(2.5) \\ 30 - 10(2.5) \end{pmatrix} = \begin{pmatrix} 37 \\ 5 \end{pmatrix} \text{ km}$$

(b) $$\underset{\sim}{r}_P - \underset{\sim}{r}_Q = \begin{pmatrix} -23 + 18(2.5) \\ 3 + 4(2.5) \end{pmatrix} - \begin{pmatrix} 37 \\ 5 \end{pmatrix} = \begin{pmatrix} 22 - 37 \\ 13 - 5 \end{pmatrix} = \begin{pmatrix} -15 \\ 8 \end{pmatrix}$$

$$\therefore |\underset{\sim}{r}_P - \underset{\sim}{r}_Q| = \left|\begin{pmatrix} -15 \\ 8 \end{pmatrix}\right| = \underline{\underline{17 \text{ km}}}$$

EXERCISE 4B

Q1. $\underset{\sim}{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -1 \end{pmatrix} = (2+5\lambda)\underset{\sim}{i} + (3-\lambda)\underset{\sim}{j}$

Q2 $\underset{\sim}{r} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix} = (3+\lambda)\underset{\sim}{i} + (\lambda - 2)\underset{\sim}{j}$

Q3. $\underset{\sim}{r} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -2 \end{pmatrix} = 5\underset{\sim}{i} + (3-2\lambda)\underset{\sim}{j}$

Q4. $\underset{\sim}{r} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -10 \end{pmatrix} = 3\lambda \underset{\sim}{i} + (5 - 10\lambda)\underset{\sim}{j}$

Q5 $\underset{\sim}{r} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 + \lambda \\ -3 + 4\lambda \end{pmatrix}$

Q6. $\underset{\sim}{r} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 5\lambda \\ 5 \end{pmatrix}$

Q7. $\underset{\sim}{b} - \underset{\sim}{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

$\therefore \underset{\sim}{r} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -4 \end{pmatrix} = (5 - 3\lambda)\underset{\sim}{i} + (3 - 4\lambda)\underset{\sim}{j}$

Q8 $\underset{\sim}{b} - \underset{\sim}{a} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ 7 \end{pmatrix} = \begin{pmatrix} -11 \\ -5 \end{pmatrix}$

$\therefore \underset{\sim}{r} = \begin{pmatrix} 6 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} -11 \\ -5 \end{pmatrix} = (6 - 11\lambda)\underset{\sim}{i} + (7 - 5\lambda)\underset{\sim}{j}$

Q9. $\underset{\sim}{b} - \underset{\sim}{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} -6 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$

$\therefore \underset{\sim}{r} = \begin{pmatrix} -6 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 8 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 + 8\lambda \\ 3 + \lambda \end{pmatrix}$

Q10. $\underset{\sim}{b} - \underset{\sim}{a} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$

$\therefore \underset{\sim}{r} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 - 4\lambda \\ -3 + 4\lambda \end{pmatrix}$

Q11 $\underset{\sim}{b} - \underset{\sim}{a} = \begin{pmatrix} -1 \\ 9 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$\therefore \underset{\sim}{r} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 - 2\lambda \\ 4 + 5\lambda \end{pmatrix}$

Q12 $\underset{\sim}{b} - \underset{\sim}{a} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$

$\therefore \underset{\sim}{r} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -6 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 - 6\lambda \\ -4\lambda \end{pmatrix}$

Q13. $\vec{OA} \Rightarrow \underset{\sim}{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

$\vec{OB} \Rightarrow \underset{\sim}{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\vec{OC} \Rightarrow \underset{\sim}{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

a) $\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix} = 2\underset{\sim}{i} - 8\underset{\sim}{j}$

b) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$
$= \begin{pmatrix} 4 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix}$
$= \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$|\overrightarrow{BC}| = \left|\begin{pmatrix} 1 \\ -4 \end{pmatrix}\right|$
$= \sqrt{1+16}$
$= \sqrt{17}$

c) $\overrightarrow{AB} : \overrightarrow{BC}$
$\begin{pmatrix} 2 \\ -8 \end{pmatrix} : \begin{pmatrix} 1 \\ -4 \end{pmatrix}$
$2\begin{pmatrix} 1 \\ -4 \end{pmatrix} : \begin{pmatrix} 1 \\ -4 \end{pmatrix}$
$2 : 1$

Q14

a) $\underset{\sim}{r} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 7 \\ 2 \end{pmatrix}$
$= (5+7\lambda)\underset{\sim}{i} + (2\lambda-1)\underset{\sim}{j}$

b) $x = 5+7\lambda$
$y = 2\lambda - 1$

c) $\lambda = \dfrac{y+1}{2}$

$\therefore x = 5 + 7\left(\dfrac{y+1}{2}\right)$

$2x = 10 + 7y + 7$
$y = \dfrac{2x-17}{7}$

$y = \frac{2}{7}x - \frac{17}{7}$

Q15 a) $\underset{\sim}{r} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} -3 \\ 4 \end{pmatrix}$
$= (2-3\lambda)\underset{\sim}{i} + (4\lambda-1)\underset{\sim}{j}$

b) $x = 2-3\lambda$
$y = 4\lambda - 1$

c) $\lambda = \dfrac{2-x}{3}$

$\therefore y = 4\left(\dfrac{2-x}{3}\right) - 1$

$3y = 8 - 4x - 1$
$y = -\frac{4}{3}x + \frac{7}{3}$ //

Q16. a) $\underset{\sim}{r} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 7 \\ -8 \end{pmatrix}$
$= 7\lambda\underset{\sim}{i} + (3-8\lambda)\underset{\sim}{j}$

b) $x = 7\lambda$
$y = 3 - 8\lambda$

c) $\therefore \lambda = \frac{x}{7}$
$y = 3 - 8\left(\frac{x}{7}\right)$
$y = 3 - \frac{8}{7}x$

Q17. $x = 2-3\lambda$, $y = -5+2\lambda$

a) $\underset{\sim}{r} = (2-3\lambda)\underset{\sim}{i} + (-5+2\lambda)\underset{\sim}{j}$
$\underset{\sim}{r} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} + \lambda\begin{pmatrix} -3 \\ 2 \end{pmatrix}$.

b) $2x = 4 - 6\lambda$
$3y = -15 + 6\lambda$
$2x + 3y = -11$

Q18 $\underset{\sim}{r} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 3 \end{pmatrix}$
$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix}$
$= \begin{pmatrix} 3 \\ -4 \end{pmatrix}$
$\overrightarrow{OE} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 3 \end{pmatrix}$
$= \begin{pmatrix} 0 \\ 5 \end{pmatrix}$
$\overrightarrow{OF} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + 3\begin{pmatrix} -1 \\ 3 \end{pmatrix}$
$= \begin{pmatrix} -1 \\ 8 \end{pmatrix}$

a) $\overrightarrow{EF} = \overrightarrow{OF} - \overrightarrow{OE}$
$= \begin{pmatrix} -1 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \end{pmatrix}$
$= \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

b) $\overrightarrow{ED} = \overrightarrow{OD} - \overrightarrow{OE}$
$= \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \end{pmatrix}$
$= \begin{pmatrix} 3 \\ -9 \end{pmatrix}$

c) $|\overrightarrow{DE}| = |\overrightarrow{ED}| = \left|\begin{pmatrix} 3 \\ -9 \end{pmatrix}\right|$
$= \sqrt{9+81}$
$= \sqrt{90} = 3\sqrt{10}$

d) $\overrightarrow{DE} : \overrightarrow{EF}$
$\begin{pmatrix} -3 \\ 9 \end{pmatrix} : \begin{pmatrix} -1 \\ 3 \end{pmatrix}$
$3 : 1$

e) $\overrightarrow{DE} : \overrightarrow{FE}$

$\begin{pmatrix}-3\\9\end{pmatrix} : \begin{pmatrix}1\\-3\end{pmatrix}$

$-3 : 1$

or $3 : -1$

f) $|\overrightarrow{DE}| : |\overrightarrow{FE}|$

$3\sqrt{10} : \sqrt{10}$

$\underline{\underline{3 : 1}}$

Q19 $\underset{\sim}{r} = \begin{pmatrix}7\\-2\end{pmatrix} + \lambda\begin{pmatrix}-2\\6\end{pmatrix}$

$= (7-2\lambda)\underset{\sim}{i} + (6\lambda-2)\underset{\sim}{j}$

B $\begin{pmatrix}1\\16\end{pmatrix} = \begin{pmatrix}7\\-2\end{pmatrix} + \lambda\begin{pmatrix}-2\\6\end{pmatrix}$

$\begin{pmatrix}-6\\18\end{pmatrix} = \lambda\begin{pmatrix}-2\\6\end{pmatrix}$

$\underline{\underline{\lambda = 3}}$

$\therefore$ B is on the line.

C $\begin{pmatrix}2\\13\end{pmatrix} = \begin{pmatrix}7\\-2\end{pmatrix} + \lambda\begin{pmatrix}-2\\6\end{pmatrix}$

$\begin{pmatrix}-5\\15\end{pmatrix} = \lambda\begin{pmatrix}-2\\6\end{pmatrix}$

$\underline{\lambda = 2.5}$

$\therefore$ C is on the line.

D $\begin{pmatrix}8\\-7\end{pmatrix} = \begin{pmatrix}7\\-2\end{pmatrix} + \lambda\begin{pmatrix}-2\\6\end{pmatrix}$

$\begin{pmatrix}1\\-5\end{pmatrix} = \lambda\begin{pmatrix}-2\\6\end{pmatrix}$

$-1 = 2\lambda \qquad -5 = 6\lambda$

$\lambda = \frac{-1}{2} \qquad \lambda = -\frac{5}{6}$

$\therefore$ D is not on the line.

E $\begin{pmatrix}-2\\5\end{pmatrix} = \begin{pmatrix}7\\-2\end{pmatrix} + \lambda\begin{pmatrix}-2\\6\end{pmatrix}$

$\begin{pmatrix}-9\\7\end{pmatrix} = \lambda\begin{pmatrix}-2\\6\end{pmatrix}$

$-9 = -2\lambda \qquad 7 = 6\lambda$

$\lambda = 4.5 \qquad \lambda = \frac{7}{6}$

$\therefore$ E is not on the line

Q20. $\underset{\sim}{r} = \begin{pmatrix}4\\-9\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\end{pmatrix}$

$= (4-\lambda)\underset{\sim}{i} + (2\lambda-9)\underset{\sim}{j}$

G $\begin{pmatrix}5\\9\end{pmatrix} = \begin{pmatrix}4\\-9\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\end{pmatrix}$

$\begin{pmatrix}1\\18\end{pmatrix} = \lambda\begin{pmatrix}-1\\2\end{pmatrix}$

$1 = -\lambda \qquad 18 = 2\lambda$

$\lambda = -1 \qquad \lambda = 9$

$\therefore$ G is not on the line

H $\begin{pmatrix}0\\-1\end{pmatrix} = \begin{pmatrix}4\\-9\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\end{pmatrix}$

$\begin{pmatrix}-4\\8\end{pmatrix} = \lambda\begin{pmatrix}-1\\2\end{pmatrix}$

$\lambda = 4$

$\therefore$ H is on the line.

I $\begin{pmatrix}-3\\5\end{pmatrix} = \begin{pmatrix}4\\-9\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\end{pmatrix}$

$\begin{pmatrix}-7\\14\end{pmatrix} = \lambda\begin{pmatrix}-1\\2\end{pmatrix}$

$\lambda = 7$

$\therefore$ I is on the line.

Q21 $\underset{\sim}{r} = \begin{pmatrix}3\\-1\end{pmatrix} + \lambda\begin{pmatrix}6\\8\end{pmatrix}$

A $\begin{pmatrix}-3\\a\end{pmatrix} = \begin{pmatrix}3\\-1\end{pmatrix} + \lambda\begin{pmatrix}6\\8\end{pmatrix}$

$\begin{pmatrix}-6\\a+1\end{pmatrix} = \lambda\begin{pmatrix}6\\8\end{pmatrix}$

$\therefore -6 = 6\lambda$

$\underline{\underline{\lambda = -1}}$

$\therefore a+1 = -8$

$\underline{\underline{a = -9}}$

B $\begin{pmatrix}b\\23\end{pmatrix} = \begin{pmatrix}3\\-1\end{pmatrix} + \lambda\begin{pmatrix}6\\8\end{pmatrix}$

$\begin{pmatrix}b-3\\24\end{pmatrix} = \begin{pmatrix}6\\8\end{pmatrix}\lambda$

$24 = 8\lambda$

$\underline{\underline{\lambda = 3}}$

$\therefore b-3 = 18$

$\underline{\underline{b = 21}}$

C $\begin{pmatrix}-9\\c\end{pmatrix} = \begin{pmatrix}3\\-1\end{pmatrix} + \lambda\begin{pmatrix}6\\8\end{pmatrix}$

$\begin{pmatrix}-12\\c+1\end{pmatrix} = \lambda\begin{pmatrix}6\\8\end{pmatrix}$

$-12 = 6\lambda \qquad \therefore c+1 = -16$

$\underline{\underline{\lambda = -2}} \qquad \underline{\underline{c = -17}}$

(D) $\begin{pmatrix} d \\ -21 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$\begin{pmatrix} d-3 \\ -20 \end{pmatrix} = \lambda \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$-20 = 8\lambda$

$\lambda = \frac{-20}{8}$

$\lambda = \frac{-5}{2}$

$\therefore \ d-3 = -\frac{5}{2}(6)$

$d-3 = -15$

$\underline{\underline{d = -12}}$

(E) $\begin{pmatrix} 12 \\ e \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$\begin{pmatrix} 9 \\ e+1 \end{pmatrix} = \lambda \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$9 = 6\lambda$

$\lambda = \frac{9}{6}$

$\lambda = \frac{3}{2}$

$\therefore \ e+1 = \frac{3}{2}(8)$

$e+1 = 12$

$\underline{\underline{e = 11}}$

(F) $\begin{pmatrix} f \\ f \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$\begin{pmatrix} f-3 \\ f+1 \end{pmatrix} = \lambda \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$f-3 = 6\lambda$

$- \quad f+1 = 8\lambda$

$-4 = -2\lambda$

$\underline{\underline{\lambda = 2}}$

$\therefore \ f-3 = 12$

$\underline{\underline{f = 15}}$

Q22 $r_1 = \begin{pmatrix} 2+\lambda \\ 3-\lambda \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\therefore \ r_2 = \begin{pmatrix} 5 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$= (5+\lambda)\underset{\sim}{i} + (-6-\lambda)\underset{\sim}{j}$

Q23. $r_1 = \begin{pmatrix} 2+3\lambda \\ 1-4\lambda \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

$\therefore \ r_2 = \begin{pmatrix} 6 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

$\underline{\underline{= (6+3\lambda)\underset{\sim}{i} + (5-4\lambda)\underset{\sim}{j}}}$

Q24 $\underset{\sim}{r} = \begin{pmatrix} 2 \\ 12 \end{pmatrix} + t \begin{pmatrix} 6 \\ -10 \end{pmatrix}$

$x = 2 + 6t \Rightarrow 5x = 10 + 30t$

$y = 12 - 10t \Rightarrow 3y = 36 - 30t$

$\therefore \ \underline{\underline{5x + 3y = 46}}$

Q25 $\underset{\sim}{r} = \begin{pmatrix} 2 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(A) $\begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 2+\lambda \\ 8-2\lambda \end{pmatrix}$

$8 - 2\lambda = 0$

$2\lambda = 8$

$\underline{\underline{\lambda = 4}}$

$\therefore \ a = 2 + 4 = \underline{6}$

$\therefore \ \vec{OA} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ i.e $6\underset{\sim}{i}$

(B) $\begin{pmatrix} 0 \\ b \end{pmatrix} = \begin{pmatrix} 2+\lambda \\ 8-2\lambda \end{pmatrix}$

$2 + \lambda = 0$

$\underline{\lambda = -2}$

$\therefore \ b = 8 - 2(-2)$

$\underline{b = 12}$

$\therefore \ \underline{\underline{\vec{OB} = \begin{pmatrix} 0 \\ 12 \end{pmatrix}}}$ i.e $12\underset{\sim}{j}$

Q26 $\underset{\sim}{r} = \begin{pmatrix} 5 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

(A) $\begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$0 = -4 - \lambda$

$\underline{\underline{\lambda = -4}}$

$\therefore \ a = 5 - 4(2)$

$\underline{\underline{a = -3}}$

$\therefore \ \vec{OA} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$ i.e $-3\underset{\sim}{i}$.

(B) $\begin{pmatrix} 11 \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$5 + 2\lambda = 11$

$2\lambda = 6$

$\underline{\underline{\lambda = 3}}$

$\therefore \ c = -4 - 3$

$\underline{\underline{c = -7}}$

Q27.

$\vec{OA} = \begin{pmatrix}2\\3\end{pmatrix}$

$\vec{OB} = \begin{pmatrix}b\\7\end{pmatrix}$

$\vec{OC} = \begin{pmatrix}5\\-4\end{pmatrix}$

$\vec{OD} = \begin{pmatrix}-2\\d\end{pmatrix}$

$\underset{\sim}{c} - \underset{\sim}{a} = \begin{pmatrix}5\\-4\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix}$

$= \begin{pmatrix}3\\-7\end{pmatrix}$

$\therefore \underset{\sim}{r} = \begin{pmatrix}2\\3\end{pmatrix} + \lambda\begin{pmatrix}3\\-7\end{pmatrix}$

$\therefore \begin{pmatrix}b\\7\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix} + \lambda\begin{pmatrix}3\\-7\end{pmatrix}$

$\begin{pmatrix}b-2\\4\end{pmatrix} = \lambda\begin{pmatrix}3\\-7\end{pmatrix}$

$4 = -7\lambda$

$\lambda = -\frac{4}{7}$

$\therefore b - 2 = 3\left(-\frac{4}{7}\right)$

$b = -\frac{12}{7} + 2$

$b = \frac{2}{7}$

$\begin{pmatrix}-2\\d\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix} + \lambda\begin{pmatrix}3\\-7\end{pmatrix}$

$\begin{pmatrix}-4\\d-3\end{pmatrix} = \lambda\begin{pmatrix}3\\-7\end{pmatrix}$

$-4 = 3\lambda$

$\lambda = \frac{-4}{3}$

$\therefore d - 3 = -7\left(\frac{-4}{3}\right)$

$d = \frac{28}{3} + 3$

$d = \frac{37}{3}$

Q28 $\underset{\sim}{r}_1 = \begin{pmatrix}5\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\4\end{pmatrix}$

$\underset{\sim}{r}_2 = \begin{pmatrix}9\\d\end{pmatrix} + \mu\begin{pmatrix}2\\c\end{pmatrix}$

If $\underset{\sim}{r}_1 = \underset{\sim}{r}_2$, then

- $\begin{pmatrix}9\\d\end{pmatrix}$ lies on $\underset{\sim}{r}_1$, $\begin{pmatrix}5\\3\end{pmatrix}$ lies on $\underset{\sim}{r}_2$
- $\begin{pmatrix}2\\c\end{pmatrix} = k\begin{pmatrix}1\\4\end{pmatrix}$

$\therefore \boxed{c = 8}$

$\begin{pmatrix}5\\3\end{pmatrix} = \begin{pmatrix}9\\d\end{pmatrix} + \mu\begin{pmatrix}2\\8\end{pmatrix}$

$\mu = -2$ $\quad \therefore 3 = d - 16 \Rightarrow \boxed{d = 19}$

Q29 $\underset{\sim}{r}_1 = \begin{pmatrix}1\\-3\end{pmatrix} + \lambda\begin{pmatrix}3\\4\end{pmatrix}$

$\underset{\sim}{r}_2 = \begin{pmatrix}e\\5\end{pmatrix} + \mu\begin{pmatrix}1\\f\end{pmatrix}$

If $\underset{\sim}{r}_1 = \underset{\sim}{r}_2$,

then $\begin{pmatrix}e\\5\end{pmatrix}$ is on $\underset{\sim}{r}_1$

and $\begin{pmatrix}3\\4\end{pmatrix} = k\begin{pmatrix}1\\f\end{pmatrix}$

$\therefore \boxed{f = \frac{4}{3}}$

$\begin{pmatrix}e\\5\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix} + \lambda\begin{pmatrix}3\\4\end{pmatrix}$

$5 = -3 + 4\lambda$

$8 = 4\lambda$

$\lambda = 2$

$\therefore e = 1 + 3(2)$

$\boxed{e = 7}$

Q30.

① $x = 1 + 2\lambda$

$y = \lambda + 3$

$x = 1 + 2(y - 3)$

$x = 1 + 2y - 6$

$2y = x + 5$

$y = \frac{x}{2} + \frac{5}{2}$

② $x = 2\lambda - 2$

$y = 1 + \lambda$

$x = 2(y - 1) - 2$

$x = 2y - 2 - 2$

$x + 4 = 2y$

$y = \frac{1}{2}x + 2$

③ $x = 8 + 2\lambda$

$y = 6 + \lambda$

$x = 8 + 2(y - 6)$

$= 8 + 2y - 12$

$y = \frac{x}{2} + 2$

① because ② + ③ are the same line

Q31

$L_1 : \underline{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$L_2 : \underline{r} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

If $\perp$, then

$\begin{pmatrix} -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 2 \end{pmatrix} = 0.$

$$\begin{pmatrix} -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 2 \end{pmatrix} = -6 + 6 = \underline{\underline{0}}$$

$\therefore \underline{\underline{L_1 \perp L_2}}.$

Q32. $L_1 : \underline{r}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Perpendicular $\Rightarrow \begin{pmatrix} 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = 0$

$3a + 2b = 0$

Let $\underline{a = 1}$,

$3 + 2b = 0$

$2b = -3$

$\underline{b = -\frac{3}{2}}$

(or $a = -2, b = 3$.)

$\therefore L_2 : \underline{r}_2 = \underline{\underline{\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 3 \end{pmatrix}}}$

Q33 $L_1 : \underline{r}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$L_2 : \underline{r}_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$$\cos\theta = \frac{\begin{pmatrix} 1 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\sqrt{17}\sqrt{5}} = \frac{2 - 4}{\sqrt{85}}$$

$\theta = \cos^{-1}\left(\frac{-2}{\sqrt{85}}\right)$

$\theta = 102.53°$

$\therefore$ acute angle $= 180 - 102.53$

$= \underline{\underline{77.47°}}$

$\underline{\underline{\approx 77°}}$

EXERCISE 4C

Q1 $\underline{r}_1 = \begin{pmatrix} 14 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -4 \end{pmatrix}$

$\underline{r}_2 = \begin{pmatrix} 9 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} -4 \\ 6 \end{pmatrix}$

If intersecting, then $\underline{r}_1 = \underline{r}_2$

$$\begin{pmatrix} 14 + 5\lambda \\ -1 - 4\lambda \end{pmatrix} = \begin{pmatrix} 9 - 4\mu \\ -4 + 6\mu \end{pmatrix}$$

$14 + 5\lambda = 9 - 4\mu \qquad -1 - 4\lambda = -4 + 6\mu$

$5\lambda + 4\mu = -5 \quad (1) \qquad 4\lambda + 6\mu = 3 \quad (2)$

$\Rightarrow 15\lambda + 12\mu = -15$

$-\ \ 8\lambda + 12\mu = 6$

$7\lambda = -21$

$\underline{\underline{\lambda = -3}}$

$\therefore 4(-3) + 6\mu = 3$

$6\mu = 15$

$\underline{\underline{\mu = \frac{5}{2}}}$

$\therefore \underline{r}_1 = \begin{pmatrix} 14 \\ -1 \end{pmatrix} - 3\begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \end{pmatrix}$

Check: $\underline{r}_2 = \begin{pmatrix} 9 \\ -4 \end{pmatrix} + \frac{5}{2}\begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 9 - 10 \\ -4 + 15 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -1 \\ 11 \end{pmatrix}}}$

Q2. $\underline{r}_1 = \begin{pmatrix} -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\underline{r}_2 = \begin{pmatrix} -10 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

If intersecting, $\underline{r}_1 = \underline{r}_2$

$$\begin{pmatrix} -3 + \lambda \\ 4 - \lambda \end{pmatrix} = \begin{pmatrix} -10 - 4\mu \\ 2 + \mu \end{pmatrix}$$

$-3 + \lambda = -10 - 4\mu \qquad 4 - \lambda = 2 + \mu$

$\lambda + 4\mu = -7 \quad (1) \qquad \mu + \lambda = 2 \quad (2)$

$-\ \ \lambda + \mu = 2 \quad (2)$

$3\mu = -9$

$\underline{\underline{\mu = -3}}$

$\lambda = 2 - (-3)$

$\underline{\underline{\lambda = 5}}$

$$\underset{\sim}{r}_2 = \begin{pmatrix}-10\\2\end{pmatrix} - 3\begin{pmatrix}-4\\1\end{pmatrix}$$
$$= \begin{pmatrix}2\\-1\end{pmatrix}$$

Check:
$$\underset{\sim}{r}_1 = \begin{pmatrix}-3\\4\end{pmatrix} + 5\begin{pmatrix}1\\-1\end{pmatrix}$$
$$= \begin{pmatrix}2\\-1\end{pmatrix}$$

Q3. $\underset{\sim}{r}_1 = \begin{pmatrix}-1\\0\end{pmatrix} + \lambda\begin{pmatrix}4\\-10\end{pmatrix}$

$\underset{\sim}{r}_2 = \begin{pmatrix}-5\\-9\end{pmatrix} + \mu\begin{pmatrix}1\\7\end{pmatrix}$

If intersecting, $\underset{\sim}{r}_1 = \underset{\sim}{r}_2$

$$\begin{pmatrix}-1+4\lambda\\-10\lambda\end{pmatrix} = \begin{pmatrix}-5+\mu\\-9+7\mu\end{pmatrix}$$

$-1+4\lambda = -5+\mu$ $\quad$ $-10\lambda = -9+7\mu$

$4\lambda - \mu = -4$ (1) $\quad$ $9 = 7\mu + 10\lambda$ (2)

$28\lambda - 7\mu = -28$

$+\ 10\lambda + 7\mu = 9$

$38\lambda = -19$

$\lambda = -\frac{1}{2}$

$-\mu = -4 - 4(-\frac{1}{2})$

$= -4 + 2$

$-\mu = -2$

$\mu = 2$

$\therefore \underset{\sim}{r}_1 = \begin{pmatrix}-1\\0\end{pmatrix} - \frac{1}{2}\begin{pmatrix}4\\-10\end{pmatrix}$

$= \begin{pmatrix}-3\\5\end{pmatrix}$

Check: $\underset{\sim}{r}_2 = \begin{pmatrix}-5\\-9\end{pmatrix} + 2\begin{pmatrix}1\\7\end{pmatrix}$

$= \begin{pmatrix}-3\\5\end{pmatrix}$

Q4 $\underset{\sim}{r}_A = \begin{pmatrix}16\\0\end{pmatrix} + \lambda\begin{pmatrix}3\\2\end{pmatrix}$

$\underset{\sim}{r}_B = \begin{pmatrix}-1\\6\end{pmatrix} + \mu\begin{pmatrix}2\\-3\end{pmatrix}$

$$\begin{pmatrix}16\\0\end{pmatrix} + \lambda\begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}-1\\6\end{pmatrix} + \mu\begin{pmatrix}2\\-3\end{pmatrix}$$

$16 + 3\lambda = -1 + 2\mu$ $\quad$ $2\lambda = 6 - 3\mu$

$3\lambda - 2\mu = -17$ (1) $\quad$ $2\lambda + 3\mu = 6$ (2)

$6\lambda - 4\mu = -34$

$-\ 6\lambda + 9\mu = 18$

$-13\mu = -52$

$\mu = 4$

$\therefore 2\lambda + 12 = 6$

$2\lambda = -6$

$\lambda = -3$

$\underset{\sim}{r}_A = \begin{pmatrix}16\\0\end{pmatrix} - 3\begin{pmatrix}3\\2\end{pmatrix}$

$= \begin{pmatrix}7\\-6\end{pmatrix}$

$\underset{\sim}{r}_B = \begin{pmatrix}-1\\6\end{pmatrix} + 4\begin{pmatrix}2\\-3\end{pmatrix}$

$= \begin{pmatrix}7\\-6\end{pmatrix}$

$\therefore$ Given that time is defined $t \geq 0$ and $\lambda = -3$, then the particles paths do not cross nor do the particles collide.

[If time was allowed to be negative, then particles would cross paths at $\begin{pmatrix}7\\-6\end{pmatrix}$]

Q5. $\underset{\sim}{r}_A = \begin{pmatrix}1\\4\end{pmatrix} + \lambda\begin{pmatrix}4\\1\end{pmatrix}$

$\underset{\sim}{r}_B = \begin{pmatrix}37\\-20\end{pmatrix} + \mu\begin{pmatrix}-2\\5\end{pmatrix}$

$1 + 4\lambda = 37 - 2\mu$ $\quad$ $4 + \lambda = 5\mu - 20$

$4\lambda + 2\mu = 36$ (1) $\quad$ $5\mu - \lambda = 24$ (2)

$\lambda = 5\mu - 24$

$4(5\mu - 24) + 2\mu = 36$ $\quad$ $\lambda = 5(6) - 24$

$20\mu - 96 + 2\mu = 36$ $\quad$ $\lambda = 6$

$22\mu = 132$

$\mu = 6$

$$r_A = \begin{pmatrix}1\\4\end{pmatrix} + 6\begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}25\\10\end{pmatrix}$$

$$r_B = \begin{pmatrix}37\\-20\end{pmatrix} + 6\begin{pmatrix}-2\\5\end{pmatrix} = \begin{pmatrix}25\\10\end{pmatrix}$$

$\therefore$ Particles collide when $t = 6$ at the point $\begin{pmatrix}25\\10\end{pmatrix}$.

Q6.

$$r_A = \begin{pmatrix}1\\19\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\end{pmatrix}$$

$$r_B = \begin{pmatrix}3\\8\end{pmatrix} + \mu\begin{pmatrix}3\\1\end{pmatrix}$$

$$\begin{pmatrix}1+2\lambda\\19-\lambda\end{pmatrix} = \begin{pmatrix}3+3\mu\\8+\mu\end{pmatrix}$$

$1 + 2\lambda = 3 + 3\mu \qquad 19 - \lambda = 8 + \mu$

$2\lambda - 3\mu = 2$ ① $\qquad \mu + \lambda = 11$ ②

$$\begin{aligned} 2\lambda - 3\mu &= 2 \\ -\ 2\lambda + 2\mu &= 22 \\ \hline -5\mu &= -20 \\ \mu &= 4 \end{aligned}$$

$\lambda = 11 - 4$

$\lambda = 7$

$$r_A = \begin{pmatrix}1\\19\end{pmatrix} + 7\begin{pmatrix}2\\-1\end{pmatrix} = \begin{pmatrix}15\\12\end{pmatrix}$$

$$r_B = \begin{pmatrix}3\\8\end{pmatrix} + 4\begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}15\\12\end{pmatrix}$$

$\therefore$ Particles do not collide but the paths cross at $\begin{pmatrix}15\\12\end{pmatrix}$, 7 seconds into the journey for A and 4 seconds into the journey for B.

EXERCISE 4D

Q1. $n = \begin{pmatrix}3\\4\end{pmatrix}$, $a = \begin{pmatrix}2\\3\end{pmatrix}$

$$r \cdot \begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix} \cdot \begin{pmatrix}3\\4\end{pmatrix}$$

$$r \cdot \begin{pmatrix}3\\4\end{pmatrix} = 6 + 12$$

$$r \cdot \begin{pmatrix}3\\4\end{pmatrix} = 18$$

Q2. $n = \begin{pmatrix}5\\-1\end{pmatrix}$, $a = \begin{pmatrix}-1\\7\end{pmatrix}$

$$r \cdot \begin{pmatrix}5\\-1\end{pmatrix} = \begin{pmatrix}-1\\7\end{pmatrix} \cdot \begin{pmatrix}5\\-1\end{pmatrix}$$

$$r \cdot \begin{pmatrix}5\\-1\end{pmatrix} = -5 - 7$$

$$r \cdot \begin{pmatrix}5\\-1\end{pmatrix} = -12$$

Q3. $r \cdot \begin{pmatrix}1\\2\end{pmatrix} = 12$

(A) $\begin{pmatrix}0\\6\end{pmatrix} \cdot \begin{pmatrix}1\\2\end{pmatrix} = 0 + 12 = 12$

$\therefore$ on the line.

(B) $\begin{pmatrix}6\\3\end{pmatrix} \cdot \begin{pmatrix}1\\2\end{pmatrix} = 6 + 6 = 12$

$\therefore$ On the line

(C) $\begin{pmatrix}10\\0\end{pmatrix} \cdot \begin{pmatrix}1\\2\end{pmatrix} = 10 + 0 \neq 12$

$\therefore$ Not on the line.

(D) $\begin{pmatrix}3\\6\end{pmatrix} \cdot \begin{pmatrix}1\\2\end{pmatrix} = 3 + 12 \neq 12$

$\therefore$ Not on the line.

(E) $\begin{pmatrix}-4\\8\end{pmatrix} \cdot \begin{pmatrix}1\\2\end{pmatrix} = -4 + 16 = 12$

$\therefore$ On the line

(F) $\begin{pmatrix}14\\-1\end{pmatrix} \cdot \begin{pmatrix}1\\2\end{pmatrix} = 14 - 2 = 12$

$\therefore$ On the line.

Q4. $\underset{\sim}{r} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 10$

(u) $\begin{pmatrix} u \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 10$

$2u + 6 = 10$

$2u = 4$

$\underline{\underline{u = 2}}$

(v) $\begin{pmatrix} -10 \\ v \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 10$

$-20 + 3v = 10$

$3v = 30$

$\underline{\underline{v = 10}}$

(w) $\begin{pmatrix} w \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 10$

$2w - 12 = 10$

$2w = 22$

$\underline{\underline{w = 11}}$

(x) $\begin{pmatrix} x \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 10$

$2x - 6 = 10$

$2x = 16$

$\underline{\underline{x = 8}}$

(y) $\begin{pmatrix} 5 \\ y \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 10$

$10 + 3y = 10$

$3y = 0$

$\underline{\underline{y = 0}}$

(z) $\begin{pmatrix} z \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 10$

$2z + 18 = 10$

$2z = -8$

$\underline{\underline{z = -4}}$

Q5 $\underset{\sim}{n} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ $\underset{\sim}{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

a) $\underset{\sim}{r} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

$\underset{\sim}{r} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix} = 7$

b) Let $\underset{\sim}{r} = \begin{pmatrix} x \\ y \end{pmatrix}$

$\underline{\underline{5x + 2y = 7}}$

Q6 $\underset{\sim}{n} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ $\underset{\sim}{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

a) $\underset{\sim}{r} \cdot \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$= 4 - 5$

$\underset{\sim}{r} \cdot \begin{pmatrix} 2 \\ 5 \end{pmatrix} = -1$

b) Let $\underset{\sim}{r} = \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \end{pmatrix} = -1$

$\underline{\underline{2x + 5y = -1}}$

Q7. $\underset{\sim}{r}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$\underset{\sim}{r}_2 \cdot \begin{pmatrix} 8 \\ 2 \end{pmatrix} = 5$

If $\underset{\sim}{r}_1 \parallel \underset{\sim}{r}_2$, then

$\begin{pmatrix} 1 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 2 \end{pmatrix} = 0$

$\underline{\underline{8 - 8 = 0}}$

$\therefore \underset{\sim}{r}_1 \parallel \underset{\sim}{r}_2$.

Q8. $\underset{\sim}{n} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}$ $\underset{\sim}{a} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$\underset{\sim}{r} \cdot \begin{pmatrix} 8 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 5 \end{pmatrix}$

$= -8 + 8$

$\underset{\sim}{r} \cdot \begin{pmatrix} 8 \\ 5 \end{pmatrix} = 0$

Let $\underset{\sim}{r} = \begin{pmatrix} x \\ y \end{pmatrix}$,

$\underline{\underline{8x + 5y = 0}}$

Q9. $\underset{\sim}{r}_1 = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$\underset{\sim}{r}_2 \cdot \begin{pmatrix} 6 \\ -4 \end{pmatrix} = -4$

If $\underset{\sim}{r}_1 \perp \underset{\sim}{r}_2$, then $\begin{pmatrix} 3 \\ -2 \end{pmatrix} = k \begin{pmatrix} 6 \\ -4 \end{pmatrix}$,

$3 = 6k$ $\quad -2 = -4k$

$\underline{k = \tfrac{1}{2}}$ $\quad \underline{k = \tfrac{1}{2}}$

$\underline{\underline{\therefore \underset{\sim}{r}_1 \perp \underset{\sim}{r}_2}}$

EXERCISE 4E:

Q1 a) $x = 4 + t$

$y = 2t$

$t = x - 4$

$\therefore y = 2(x-4)$

$\underline{\underline{y = 2x - 8}}$

b) $x = t$

$y = \frac{1}{t}$

$\therefore \underline{\underline{y = \frac{1}{x}}}$.

c) $x = t^2$

$y = 2t$

$t = \pm\sqrt{x}$

$\therefore \underline{\underline{y = \pm 2\sqrt{x}}}$

(or $y^2 = 4x$)

d) $x = \sqrt{t-1}$

$y = t^2$

$x^2 = t - 1$

$t = x^2 + 1$

$\therefore y = (x^2+1)^2, \ x \geq 0$.

Q2 a) $\underset{\sim}{r} = (3-t)\underset{\sim}{i} + (4+2t)\underset{\sim}{j}$

$x = 3 - t$

$y = 4 + 2t$

$t = 3 - x$

$\therefore y = 4 + 2(3-x)$

$y = 4 + 6 - 2x$

$\underline{\underline{y = 10 - 2x}}$

b) $\underset{\sim}{r} = (t-1)\underset{\sim}{i} + \frac{1}{t}\underset{\sim}{j}$

$x = t - 1$

$y = \frac{1}{t}$

$t = \frac{1}{y}$

$\therefore x = \frac{1}{y} - 1 \Rightarrow \underline{\underline{y = \frac{1}{x+1}}}$.

c) $\underset{\sim}{r} = (t-1)\underset{\sim}{i} + (t^2+4)\underset{\sim}{j}$

$x = t - 1$

$y = t^2 + 4$

$t = x + 1$

$\underline{\underline{y = (x+1)^2 + 4}}$

d) $\underset{\sim}{r} = (2+\cos\theta)\underset{\sim}{i} + (1+2\sin\theta)\underset{\sim}{j}$

$x = 2 + \cos\theta$

$y = 1 + 2\sin\theta$

$x - 2 = \cos\theta \qquad \frac{y-1}{2} = \sin^2\theta$

$(x-2)^2 + \left(\frac{y-1}{2}\right)^2 = 1$

$\underline{\underline{(x-2)^2 + \frac{(y-1)^2}{4} = 1}}$

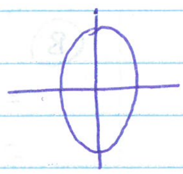
Q3. $\underset{\sim}{r} = 2\cos\theta\,\underset{\sim}{i} + 3\sin\theta\,\underset{\sim}{j}$

$x = 2\cos\theta \qquad y = 3\sin\theta$

$\frac{x}{2} = \cos\theta \qquad \frac{y}{3} = \sin\theta$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\underline{\underline{\frac{x^2}{4} + \frac{y^2}{9} = 1}}$



centred @ (0,0), horizontal radius 2, vertical radius 3.

Q4. $\underset{\sim}{r} = -3\sec\theta\,\underset{\sim}{i} + 2\tan\theta\,\underset{\sim}{j}$

$x = -3\sec\theta \qquad y = 2\tan\theta$

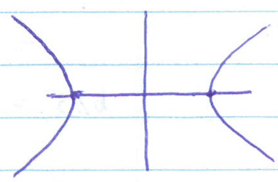
$\left(\frac{x}{-3}\right) = \sec\theta \qquad \frac{y}{2} = \tan\theta$

$\frac{x^2}{9} = \sec^2\theta \qquad \frac{y^2}{4} = \tan^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

$\frac{y^2}{4} + 1 = \frac{x^2}{9}$

$\frac{x^2}{9} - \frac{y^2}{4} = 1$



centred @ (0,0)

Q5.

A: point ✗.

B: circle ✓

All points of distance 6 from $0\underset{\sim}{i} + 0\underset{\sim}{j}$

C: Point ✗

D: circle ✓

All points of distance 24 from $5\underset{\sim}{i} - 4\underset{\sim}{j}$

E: circle ✓

$(x+2)^2 - 4 + (y-4)^2 - 16 = 5$

$(x+2)^2 + (y-4)^2 = 25$

All points 5 units from $(-2, 4)$.

F: not a circle. ✗

Q6. a) $|\underset{\sim}{r}| = 25$

b) $|\underset{\sim}{r}|^2 = 625$

A $\left|\begin{pmatrix} 19 \\ -18 \end{pmatrix}\right|^2$

$= 685$

$685 > 625$

$\therefore$ outside the circle

B $\left|\begin{pmatrix} -20 \\ 15 \end{pmatrix}\right|^2$

$= 400 + 225$

$= 625$

$625 = 625$

$\therefore$ On the circle.

C: $\left|\begin{pmatrix} 14 \\ 17 \end{pmatrix}\right|^2$

$= 14^2 + 17^2$

$= 485$

$485 < 625$

$\therefore$ Inside the circle.

D $\left|\begin{pmatrix} -24 \\ -7 \end{pmatrix}\right|^2$

$= 576 + 49$

$= 625$

$625 = 625$

$\therefore$ On the circle.

Q7. $|\underset{\sim}{r}| = 65$

$x^2 + y^2 = 65^2$

$x^2 + y^2 = 4225$

$(-52)^2 + a^2 = 4225$

$a^2 = 1521$

$a = \pm 39$

$\therefore a = 39$ as $a > 0$.

$b^2 + 25^2 = 4225$

$b^2 = 3600$

$a = \pm 60$

$\therefore a = -60$ as $b < 0$.

Q8

$\left|\underset{\sim}{r} - \begin{pmatrix} -7 \\ 4 \end{pmatrix}\right| = 4\sqrt{5}$

LHS) $\left|\underset{\sim}{r} - \begin{pmatrix} -7 \\ 4 \end{pmatrix}\right|$

$= \left|\begin{pmatrix} 1 \\ 8 \end{pmatrix} - \begin{pmatrix} -7 \\ 4 \end{pmatrix}\right|$

$= \left|\begin{pmatrix} 8 \\ 4 \end{pmatrix}\right|$

$= \sqrt{64+16}$

$= \sqrt{80}$

$= 4\sqrt{5}$

$= \text{RHS}$

$\therefore$ Point lies on the circle.

Q9.

a) $\left|\underset{\sim}{r} - \begin{pmatrix} 1 \\ -5 \end{pmatrix}\right| = 9$

b) $\left|\underset{\sim}{r} - \begin{pmatrix} -3 \\ 4 \end{pmatrix}\right| = 10$

c) $\left|\underset{\sim}{r} - \begin{pmatrix} -12 \\ 3 \end{pmatrix}\right| = 2\sqrt{3}$

d) $\left|\underset{\sim}{r} - \begin{pmatrix} -13 \\ -2 \end{pmatrix}\right| = 4.$

Q10 a) $\left|\underset{\sim}{r} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right| = 5$

Let $\underset{\sim}{r} = \begin{pmatrix} x \\ y \end{pmatrix}$.

$\left|\begin{pmatrix} x-2 \\ y-3 \end{pmatrix}\right| = 5$

$(x-2)^2 + (y-3)^2 = 25$

$x^2-4x+4+y^2-6y+9=25$

$x^2-4x+y^2-6y=12$

$\underline{x^2+y^2-4x-6y=12}$

b) $|\underset{\sim}{r}-\begin{pmatrix}-4\\2\end{pmatrix}|=\sqrt{7}$

Let $\underset{\sim}{r}=\begin{pmatrix}x\\y\end{pmatrix}$.

$\left|\begin{pmatrix}x+4\\y-2\end{pmatrix}\right|=\sqrt{7}$

$(x+4)^2+(y-2)^2=7$

$x^2+8x+16+y^2-4y+4=7$

$x^2+y^2+8x-4y=-13$

c) $|\underset{\sim}{r}-\begin{pmatrix}4\\-3\end{pmatrix}|=7$

Let $\underset{\sim}{r}=\begin{pmatrix}x\\y\end{pmatrix}$,

$\left|\begin{pmatrix}x-4\\y+3\end{pmatrix}\right|=7$

$(x-4)^2+(y+3)^2=49$

$x^2-8x+16+y^2+6y+9=49$

$x^2+y^2-8x+6y=24.$

Q11.

a) centre @ $6\underset{\sim}{i}+3\underset{\sim}{j}$

radius, $r=5$

b) $|\underset{\sim}{r}-\begin{pmatrix}2\\-3\end{pmatrix}|=6$

centre @ $2\underset{\sim}{i}-3\underset{\sim}{j}$

radius, $r=6$.

c) $|(x-3)\underset{\sim}{i}+(y+4)\underset{\sim}{j}|=3$

$\left|\begin{pmatrix}x\\y\end{pmatrix}-\begin{pmatrix}3\\-4\end{pmatrix}\right|=3$

$|\underset{\sim}{r}-\begin{pmatrix}3\\-4\end{pmatrix}|=3$

centre @ $3\underset{\sim}{i}-4\underset{\sim}{j}$

radius, $r=3$.

d) centre @ $0\underset{\sim}{i}+0\underset{\sim}{j}$

radius, $r=20$.

e) $16x^2+16y^2=25$

$x^2+y^2=\frac{25}{16}$

$\therefore$ centre @ $0\underset{\sim}{i}+0\underset{\sim}{j}$

radius, $r=\frac{5}{4}$

f) $(x-2)^2+(y+3)^2=49$

centre @ $2\underset{\sim}{i}-3\underset{\sim}{j}$

radius, $r=7$.

g) $x^2+y^2-6x-18y+65=0$

$(x-3)^2-9+(y-9)^2-81+65=0$

$(x-3)^2+(y-9)^2=25$

centre @ $3\underset{\sim}{i}+9\underset{\sim}{j}$

radius, $r=5$.

h) $x^2+y^2+20x-2y=20$

$(x+10)^2-100+(y-1)^2-1=20$

$(x+10)^2+(y-1)^2=121$

centre @ $-10\underset{\sim}{i}+\underset{\sim}{j}$

radius, $r=11$.

Q12. centres @ $\begin{pmatrix}1\\-1\end{pmatrix}$ and $\begin{pmatrix}6\\11\end{pmatrix}$

$\therefore \left|\begin{pmatrix}1\\-1\end{pmatrix}-\begin{pmatrix}6\\11\end{pmatrix}\right|$

$=\left|\begin{pmatrix}-5\\-12\end{pmatrix}\right|$

$=\sqrt{25+144}$

$=\underline{13\text{ units}}$

Q13. $|\underset{\sim}{r}-\begin{pmatrix}2\\-5\end{pmatrix}|=5$

$|\underset{\sim}{r}-\begin{pmatrix}5\\2\end{pmatrix}|=3$

$\begin{pmatrix}5\\2\end{pmatrix}-\begin{pmatrix}2\\-5\end{pmatrix}=\begin{pmatrix}3\\7\end{pmatrix}$

$\therefore \underset{\sim}{r}_{AB}=\begin{pmatrix}2\\-5\end{pmatrix}+\lambda\begin{pmatrix}3\\7\end{pmatrix}$

Q14 A @ $\begin{pmatrix}3\\-2\end{pmatrix}$, radius 3

B @ $\begin{pmatrix}9\\6\end{pmatrix}$, radius 7.

$\left|\begin{pmatrix}9\\6\end{pmatrix}-\begin{pmatrix}3\\-2\end{pmatrix}\right|=\left|\begin{pmatrix}6\\8\end{pmatrix}\right|$

$=\sqrt{100}$

$=10$ units

$\therefore$ Circles are tangential, i.e 1 pt in common.

Q15 A @ $\begin{pmatrix}3\\-1\end{pmatrix}$, radius 3

B @ $\begin{pmatrix}13\\1\end{pmatrix}$, radius 7.

$\left|\begin{pmatrix}13\\1\end{pmatrix}-\begin{pmatrix}3\\-1\end{pmatrix}\right|=\left|\begin{pmatrix}10\\2\end{pmatrix}\right|$

$=\sqrt{100+4}$

$=\underline{\sqrt{104}}$ units

$\sqrt{104}>10$ $\therefore$ circles are non-intersecting.

Q16.

$$\underset{\sim}{r}_L = \begin{pmatrix}-10\\15\end{pmatrix} + \lambda\begin{pmatrix}7\\-3\end{pmatrix} = \begin{pmatrix}7\lambda-10\\15-3\lambda\end{pmatrix}$$

$$\left|\begin{pmatrix}7\lambda-10\\15-3\lambda\end{pmatrix} - \begin{pmatrix}-1\\7\end{pmatrix}\right| = \sqrt{29}$$

$$\left|\begin{pmatrix}7\lambda-9\\8-3\lambda\end{pmatrix}\right| = \sqrt{29}$$

$$(7\lambda-9)^2 + (8-3\lambda)^2 = 29$$

$$49\lambda^2 - 126\lambda + 81 + 64 - 48\lambda + 9\lambda^2 = 29$$

$$58\lambda^2 - 174\lambda + 116 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda-2)(\lambda-1) = 0$$

$\lambda = 2$ and $\lambda = 1$

$$\therefore \underset{\sim}{r}_1 = \begin{pmatrix}-10\\15\end{pmatrix} + 2\begin{pmatrix}7\\-3\end{pmatrix} = \begin{pmatrix}4\\9\end{pmatrix}$$

and

$$\underset{\sim}{r}_2 = \begin{pmatrix}-10\\15\end{pmatrix} + 1\begin{pmatrix}7\\-3\end{pmatrix} = \begin{pmatrix}-3\\12\end{pmatrix}$$

Q17. $\underset{\sim}{r} = \begin{pmatrix}10\\-9\end{pmatrix} + \lambda\begin{pmatrix}4\\-5\end{pmatrix}$

$$\left|r - \begin{pmatrix}-7\\2\end{pmatrix}\right| = \sqrt{41}$$

$$\left|\begin{pmatrix}10+4\lambda\\-9-5\lambda\end{pmatrix} - \begin{pmatrix}-7\\2\end{pmatrix}\right| = \sqrt{41}$$

$$\left|\begin{pmatrix}17+4\lambda\\-11-5\lambda\end{pmatrix}\right| = \sqrt{41}$$

$$(17+4\lambda)^2 + (-11-5\lambda)^2 = 41$$

$$289 + 136\lambda + 16\lambda^2 + 121 + 110\lambda + 25\lambda^2 = 41$$

$$41\lambda^2 + 246\lambda + 410 = 41$$

$$\lambda^2 + 6\lambda + 10 = 1$$

$$\lambda^2 + 6\lambda + 9 = 0$$

$$(\lambda+3)^2 = 0$$

$$\lambda = -3$$

$$\therefore \underset{\sim}{r} = \begin{pmatrix}10\\-9\end{pmatrix} + 3\begin{pmatrix}4\\-5\end{pmatrix} = \begin{pmatrix}-2\\6\end{pmatrix}$$

EXERCISE 4F:

Q1.

$\underset{\sim}{r}_p = \begin{pmatrix}25\\15\end{pmatrix}$ $\quad \underset{\sim}{r}_s = \lambda\begin{pmatrix}10\\5\end{pmatrix}$

$$\therefore \vec{SP} = \vec{SO} + \vec{OP} = \begin{pmatrix}25\\15\end{pmatrix} - \lambda\begin{pmatrix}10\\5\end{pmatrix}$$

$$\vec{SP}\cdot\vec{OP} = \begin{pmatrix}25-10\lambda\\15-5\lambda\end{pmatrix}\cdot\begin{pmatrix}25\\15\end{pmatrix}$$

$$0 = 625 - 250\lambda + 225 - 75\lambda = 850 - 325\lambda$$

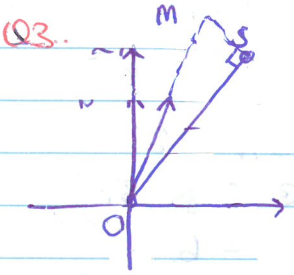
$$\lambda = \frac{850}{325}$$

$$\lambda = \frac{34}{13}$$

$$|\vec{SP}| = \left|\begin{pmatrix}25\\15\end{pmatrix} - \frac{34}{13}\begin{pmatrix}10\\5\end{pmatrix}\right| = 2.24 \text{ km}$$

at approximately 10:36 am

Q3.



$\vec{OM} = \lambda\begin{pmatrix}1\\2\end{pmatrix}$ $\quad \vec{OS} = \begin{pmatrix}5\\6\end{pmatrix}$

$$\vec{MS} = \vec{OS} - \vec{OM} = \begin{pmatrix}5\\6\end{pmatrix} - \lambda\begin{pmatrix}1\\2\end{pmatrix}$$

$$\vec{OS}\cdot\vec{MS} = \begin{pmatrix}5\\6\end{pmatrix}\cdot\begin{pmatrix}5-\lambda\\6-2\lambda\end{pmatrix}$$

$$0 = 25 - 5\lambda + 36 - 12\lambda = 61 - 17\lambda$$

$$\lambda = \frac{61}{17}$$

$$|\vec{MS}| = \left|\begin{pmatrix}5\\6\end{pmatrix} - \frac{61}{17}\begin{pmatrix}1\\2\end{pmatrix}\right| = 1.84 \text{ m}$$

$\therefore$ Snake is likely to catch mouse.

Q5

$\underset{\sim}{r}_A = \begin{pmatrix} 30 \\ 10 \end{pmatrix} + t\begin{pmatrix} 10 \\ -5 \end{pmatrix}$

$\underset{\sim}{r}_B = \begin{pmatrix} 54 \\ -19 \end{pmatrix} + t\begin{pmatrix} -8 \\ 7 \end{pmatrix}$

$|\underset{\sim}{r}_A - \underset{\sim}{r}_B| = \left|\begin{pmatrix} 30+10t \\ 10-5t \end{pmatrix} - \begin{pmatrix} 54-8t \\ -19+7t \end{pmatrix}\right|$

$= \left|\begin{pmatrix} -24+18t \\ 29-12t \end{pmatrix}\right|$

$d = \sqrt{(-24+18t)^2 + (29-12t)^2}$

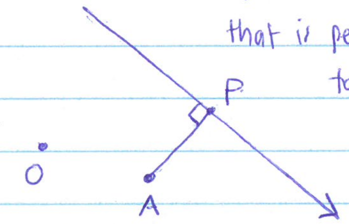
Using the CASIO,

$d' = 0$

$\Rightarrow t = \frac{5}{3}$ (ie. 4:40am)

when $t = \frac{5}{3}$, $d = \underline{\underline{3\sqrt{13}\text{ km}}}$

Q7. Let P be the point on the line that is perpendicular to A.



$\overrightarrow{OP} = \begin{pmatrix} -5 \\ 22 \end{pmatrix} + \lambda\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$\overrightarrow{OA} = \begin{pmatrix} 14 \\ -3 \end{pmatrix}$

$\therefore \overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$

$= \begin{pmatrix} -5+5\lambda \\ 22-2\lambda \end{pmatrix} - \begin{pmatrix} 14 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -19+5\lambda \\ 25-2\lambda \end{pmatrix}$

Line L is in the direction of $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$\therefore \begin{pmatrix} 5 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -19+5\lambda \\ 25-2\lambda \end{pmatrix} = 0$

$-95 + 25\lambda - 50 + 4\lambda = 0$

$29\lambda - 145 = 0$

$29\lambda = 145$

$\underline{\underline{\lambda = 5}}$

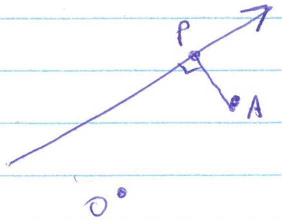
$\therefore \overrightarrow{AP}\big|_{\lambda=5} = \begin{pmatrix} 6 \\ 15 \end{pmatrix}$

$|\overrightarrow{AP}| = \sqrt{36+225}$

$= \sqrt{261}$

$= 3\sqrt{29} = \underline{\underline{16.16\text{ units (2dp)}}}$

Q8 Let P be the point on the line perpendicular to A.



$\overrightarrow{OP} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\overrightarrow{OA} = \begin{pmatrix} 11 \\ 18 \end{pmatrix}$

$\overrightarrow{AP} = \begin{pmatrix} 3+3\lambda \\ -1+4\lambda \end{pmatrix} - \begin{pmatrix} 11 \\ 18 \end{pmatrix}$

$= \begin{pmatrix} -8+3\lambda \\ -19+4\lambda \end{pmatrix}$

Line L is in the direction of $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\therefore \begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -8+3\lambda \\ -19+4\lambda \end{pmatrix} = 0$

$-24 + 9\lambda - 76 + 16\lambda = 0$

$25\lambda - 100 = 0$

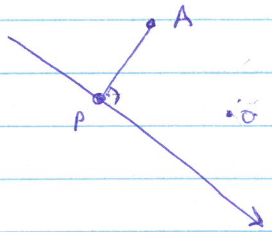
$\underline{\underline{\lambda = 4}}$

$\therefore \overrightarrow{AP}\big|_{\lambda=4} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$\therefore |\overrightarrow{AP}| = \sqrt{16+9}$

$= \underline{\underline{5\text{ units}}}$

Q9. Let P be the point on the line perpendicular to A.



$\overrightarrow{OP} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$\overrightarrow{OA} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$

$\overrightarrow{AP} = \begin{pmatrix} -3+2\lambda \\ -2\lambda \end{pmatrix} - \begin{pmatrix} -3 \\ 8 \end{pmatrix}$

$= \begin{pmatrix} 2\lambda \\ -2\lambda-8 \end{pmatrix}$

Line L is in the direction of $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$\begin{pmatrix} 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2\lambda \\ -2\lambda-8 \end{pmatrix} = 0$

$4\lambda + 4\lambda + 16 = 0$

$8\lambda = -16$

$\underline{\underline{\lambda = -2}}$

$\therefore \overrightarrow{AP}\big|_{\lambda=-2} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}$

$\therefore |\overrightarrow{AP}| = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} = \underline{\underline{5.66\text{ units}}}$